HOMEWORK 1

(1) Show that two charts being related is **not** an equivalence relation.

(2) How many distinct smooth structures are there in $\mathbb{R}$? (There's only one **up to diffeomorphism**, but that's not the question being asked)

(3) Let $X = \{(x, y) \in \mathbb{R}^2 : y = \pm 1\}$, and let M be the quotient of X by the equivalence relation generated by $(x, -1) \sim (x, 1)$ for all $x \neq 0$. Show that M is locally Euclidean and second countable, but not Hausdorff. [This space is called the line with two origins.]

(4) Compute the coordinate representation for each of the following maps, using stereographic coordinates for spheres; use this to conclude that each map is smooth.
   (a) $A : \mathbb{S}^n \to \mathbb{S}^n$ is the antipodal map $A(x) = -x$.
   (b) $F : \mathbb{S}^3 \to \mathbb{S}^2$ is given by $F(z, w) = (z\bar{w} + w\bar{z}, iw\bar{z} - iz\bar{w}, z\bar{z} - w\bar{w})$, where we think of $\mathbb{S}^3$ as the subset $\{(w, z) : w^2 + z^2 = 1\}$ of $\mathbb{C}^2$.

(5) Let C be a curve in $\mathbb{R}^3$. Show that the collection of all vectors normal to C form a $3-$dimensional manifold.