Exam I

Choose (only) 5 questions:

1. Assume that n is a positive integer. Prove that if one selects any $n+1$ numbers from the $\{1, 2, \ldots, 2n\}$, then two of the selected numbers will sum to $2n+1$.
2. Find your own real-world example of the pigeonhole principle.
3. Give an example of 100 numbers from $\{1, 2, \ldots, 200\}$ where not one of your numbers divides another.
4. Prove that any set of seven integers contains a pair whose sum or difference is divisible by 10.
5. If m and n are integers, then $7m - 3n$ is even.
6. Prove that for every integer n, either $n^2 \equiv 0 \pmod 4$ or $n^2 \equiv 1 \pmod 4$
7. Prove that n is even if and only if n^2 is even. (You'll have to prove two things: if n is even then n^2 is even; and conversely, if n^2 is even then n is even.)
8. Show that the following conjecture is false by finding a counter-example: *Let* $f(n) = n^2 - n + 5$. *For any natural number* $n \in \mathbb{N}$, $f(n)$ *is prime.*