

### Exam I

Choose (only) 5 questions:

1. Find the cardinality of the the set  $\{\{1\}, 3, \{\{1\}, 3\}\}$
2. For sets  $A, B$  and  $C$ , and a universal set  $U$ , draw the Venn diagram representing the following set  $A^c \cap (B - C)$
3. Let  $\mathcal{P}(A)$  denotes the power set of  $A$ . Show that

$$\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$$

4. Using induction prove that

$$\frac{1}{2!} + \frac{2}{3!} + \dots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

5. Using induction prove that if a set  $A$  has  $n$  elements then the power set  $\mathcal{P}(A)$  has  $2^n$  elements.
6. Rewrite the following sentence to be of the form “If P, then Q.”: *An integer is even provided it is not odd.* Make sure your new wording does not change its meaning.
7. Negate the following sentence: *If I don't pass Intro to Proofs and Linear Algebra this semester, than I will take Real Analysis and Abstract algebra next semester.*
8. Translate the following to plain English, and then write down whether it is true or false:

$$\exists n \in \mathbb{N} : \forall y \in \mathbb{Z}, x \leq y^2$$